ENERGY AND MOMENTUM OF THE SZEKERES UNIVERSES IN TELE-PARALLEL GRAVITY

S. AYGÜN†, İ. TARHAN†, H. BAYSAL‡

†Department of Physics, Art and Science Faculty, Çanakkale Onsekiz Mart University, 17020 Çanakkale, Turkey

†Çanakkale Onsekiz Mart University, Astrophysics Research Center, 17020 Çanakkale, Turkey

‡Department of Secondary Science and Mathematics Education, Faculty of Education, Çanakkale Onsekiz Mart University, 17020 Çanakkale, Turkey

In order to evaluate the energy distribution (due to matter and fields including gravitation) associated with a space-time model of Szekeres class I and II metrics, we consider the Einstein, Bergmann-Thomson and Landau-Lifshitz energy and/or momentum definitions in the tele-parallel gravity (the tetrad theory of gravitation). We find the same energy distribution using Einstein and Bergmann-Thomson formulations, but we also find that the energy-momentum prescription of Landau-Lifshitz disagree in general with these definitions. This results are the same as a previous works of Aygün *et al.*, they investigated the same problem in general relativity by using Einstein, Bergman-Thomson, Møller and Landau-Lifshitz (LL) energy-momentum complexes.

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1. Introduction

The issue of energy localization was first discussed during the early years after the development of general relativity and debate continued for decades. There are different attempts to find a general accepted definition of the energy density for the gravitational field. However, there is still no generally accepted definition known. The foremost endeavor was made by Einstein [1] who suggested a definition for energy-momentum distribution. Following this definition, many physicists proposed different energy-momentum complexes: e.g. Tolman [2], Landau and Lifshitz [3], Papapetrou [4], Bergmann and Thomson [5], Weinberg [6], Qadir and Sharif [7] and Møller [8]. Except for the Møller definition, others are restricted to calculate the energy and momentum distributions in quasi-Cartesian coordinates to get a reasonable and meaningful result.

Despite these drawbacks, some interesting results obtained recently leads to the conclusion that these definitions give exactly the same energy distribution for any given space-time [9]-[27]. However, some examples of space-times have been explored which do not support these results [28]-[32].

The problem of energy-momentum localization can also be reformulated in the context of tele-parallel gravity [33, 34, 35]. By working in the context of tele-parallel gravity, Vargas [35] obtained the tele-parallel version of both Einstein and Landau-Lifshitz energy-momentum complexes. He used these definitions and found that the total energy is zero in Friedmann-Robertson-Walker space-time. His results are the same as those calculated in general relativity. Salti and his collaborators [36]-[39] considered different space-times for various definitions in tele-parallel gravity to obtain the energy-momentum distribution in a given model. Their results agree with the previous results obtained in the theory of general relativity.

The basic purpose of this paper is that using the energy-momentum definitions of Einstein, Bergmann-Thomson and Landau-Lifshitz in tele-parallel gravity to obtain the total energy associated with Szekeres type I and II space times. We will proceed according to the following scheme. In the next section,we briefly present the Szekeres Universes. Then, in Sec. III, we give Szekeres type II space-time and its tetrad components. In Sec. IV, we give Szekeres type I space-time and its tetrad components. In Sec. V, we present the energy-momentum definitions of Einstein, Bergmann-Thomson and Landau-Lifshitz in tele-parallel gravity. In Sec. VI gives us the Szekeres class I and class II solutions. Finally, Sec. VII is devoted to concluding remarks. Throughout this paper we choose units such that $G = 1$ and $c = 1$ and follow the convention that indices take values from 0 to 3 otherwise stated.

2. The Szekeres Class I and Szekeres Class II Space-Times

Szekeres [40] derived a remarkable set of inhomogeneous exact solutions of Einstein's field equations without cosmological constant. The source of curvature of the models is an expanding, irrotational, and geodesic dust. These solutions are divided into two classes usually denoted by I and II. The class I solutions are usually presented in a way that is formally analogous to the Tolman-Bondi spherically-symmetric solutions, which they generalize. This class of solutions has primarily been used to model non-spherical collapse of an inhomogeneous dust cloud [41]. The class II solutions are usually considered as generalizations of the Kantowski-Sachs [42] and Friedmann-

Robertson -Walker (FRW) solutions and have primarily been studied as cosmological models [43]. Those of class II are more important as cosmological models, because they can closely approximate, over a finite time interval, the FRW dust models.

In this section, we introduce the Szekeres class II and Szekeres class I metrics and then using these space-times we make some required calculations.

3. The Szekeres Class II Model

The Szekeres class II space-time is defined by the line element [44]

$$ds^2 = -dt^2 + Q^2 dx^2 + R^2(dy^2 + h^2 dz^2). \tag{1}$$

where $Q=Q(x,y,z,t), R=R(t)$ and $h=h(y)$ are functions to be determined. For the line element (1), $g_{\mu\nu}$ is defined by

$$g_{\mu\nu} = -\delta^0_\mu \delta^0_\nu + Q^2 \delta^1_\mu \delta^1_\nu + R^2 \delta^2_\mu \delta^2_\nu + (Rh)^2 \delta^3_\mu \delta^3_\nu \tag{2}$$

and its inverse $g^{\mu\nu}$

$$g^{\mu\nu} = -\delta^\mu_0 \delta^\nu_0 + Q^{-2} \delta^\mu_1 \delta^\nu_1 + R^{-2} \delta^\mu_2 \delta^\nu_2 + (Rh)^{-2} \delta^\mu_3 \delta^\nu_3 \tag{3}$$

The non-trivial tetrad field induces a tele-parallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu \tag{4}$$

Using this relation, we obtain the tetrad components:

$$h^a{}_\mu = \delta^a_0 \delta^0_\mu + Q \delta^a_1 \delta^1_\mu + R \delta^a_2 \delta^2_\mu + Rh \delta^a_3 \delta^3_\mu \tag{5}$$

and its inverse is

$$h_a{}^\mu = \delta^0_a \delta^\mu_0 + Q^{-1} \delta^1_a \delta^\mu_1 + R^{-1} \delta^2_a \delta^\mu_2 + (Rh)^{-1} \delta^3_a \delta^\mu_3 \tag{6}$$

For the line-element which describes the Szekeres Type II space universe, one can introduce the tetrad basis

$$\theta^0 = dt, \qquad \theta^1 = Q dx, \qquad \theta^2 = R dy, \qquad \theta^3 = (Rh) dz. \tag{7}$$

4. The Szekeres Class I Model

The Szekeres class I space-time is defined by the line element

$$ds^2 = -dt^2 + e^{2B}(dx^2 + dy^2) + e^{2A}dz^2. \tag{8}$$

where $A = A(x, y, z, t)$, $B = B(x, y, z, t)$ are functions to be determined.

$$g_{\mu\nu} = -\delta^0_\mu \delta^0_\nu + e^{2B}\delta^1_\mu \delta^1_\nu + e^{2B}\delta^2_\mu \delta^2_\nu + e^{2A}\delta^3_\mu \delta^3_\nu \tag{9}$$

and its inverse $g^{\mu\nu}$

$$g^{\mu\nu} = -\delta^\mu_0 \delta^\nu_0 + e^{-2B}\delta^\mu_1 \delta^\nu_1 + e^{-2B}\delta^\mu_2 \delta^\nu_2 + e^{-2A}\delta^\mu_3 \delta^\nu_3 \tag{10}$$

The non-trivial tetrad field induces a tele-parallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu \tag{11}$$

Using this relation, we obtain the tetrad components:

$$h^a{}_\mu = \delta^a_0 \delta^0_\mu + e^B \delta^a_1 \delta^1_\mu + e^B \delta^a_2 \delta^2_\mu + e^A \delta^a_3 \delta^3_\mu \tag{12}$$

and its inverse is

$$h_a{}^\mu = \delta^0_a \delta^\mu_0 + e^{-B}\delta^1_a \delta^\mu_1 + e^{-B}\delta^2_a \delta^\mu_2 + e^{-A}\delta^3_a \delta^\mu_3 \tag{13}$$

For the line-element which describes the Szekeres Type I space universe, one can introduce the tetrad basis

$$\theta^0 = dt, \qquad \theta^1 = e^B dx, \qquad \theta^2 = e^B dy, \qquad \theta^3 = e^A dz. \tag{14}$$

5. Energy-Momentum in Tele-parallel Gravity

The tele-parallel gravity is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry [45]. In the theory of the tele-parallel gravity, gravitation is attributed to torsion [46], which plays the role of a force [47], and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a tele-parallel structure which is directly related to the presence of the gravitational field. The interesting place of tele-parallel gravity is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the situation, for example, in the energy and momentum problem, which becomes more transparent when considered from the tele-parallel point of view.

The Einstein, Bergmann-Thomson and Landau-Lifshitz's energy-momentum complexes in tele-parallel gravity[48] are respectively:

$$hE^{\mu}{}_{\nu} = \frac{1}{4\pi}\partial_{\lambda}(U_{\nu}{}^{\mu\lambda}) \tag{15}$$

$$hB^{\mu\nu} = \frac{1}{4\pi}\partial_{\lambda}(g^{\mu\beta}U_{\beta}{}^{\nu\lambda}) \tag{16}$$

$$hL^{\mu\nu} = \frac{1}{4\pi}\partial_{\lambda}(hg^{\mu\beta}U_{\beta}{}^{\nu\lambda}) \tag{17}$$

where $U_{\beta}{}^{\nu\lambda}$ is the Freud's super-potential, which is given by:

$$U_{\beta}{}^{\nu\lambda} = hS_{\beta}{}^{\nu\lambda}. \tag{18}$$

where $h = \mathtt{det}(h^{a}{}_{\mu})$ and $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = m_1 T^{\mu\nu\lambda} + \frac{m_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{m_3}{2}(g^{\mu\lambda}T^{\beta\nu}_{\ \ \beta} - g^{\nu\mu}T^{\beta\lambda}_{\ \ \beta}) \tag{19}$$

with m_1, m_2 and m_3 the three dimensionless coupling constants of tele-parallel gravity [49]. For the tele-parallel equivalent of general relativity the specific choice of these three constants are:

$$m_1 = \frac{1}{4}, \qquad m_2 = \frac{1}{2}, \qquad m_3 = -1 \tag{20}$$

To calculate this tensor, firstly we must calculate Weitzenböck connection:

$$\Gamma^{\alpha}{}_{\mu\nu} = h_{a}{}^{\alpha}\partial_{\nu}h^{a}{}_{\mu} \tag{21}$$

and torsion of the Weitzenböck connection:

$$T^{\mu}{}_{\nu\lambda} = \Gamma^{\mu}{}_{\lambda\nu} - \Gamma^{\mu}{}_{\nu\lambda} \tag{22}$$

The energy-momentum complexes of Einstein, Bergmann-Thomson and Landau-Lifshitz in the tele-parallel gravity are given by the following equations, respectively,

$$P_{\mu}^{E} = \int_{\Sigma} hE^{0}{}_{\mu}dxdydz, \tag{23}$$

$$P_{\mu}^{B} = \int_{\Sigma} hB^{0}{}_{\mu}dxdydz, \tag{24}$$

$$P_\mu^L = \int_\Sigma h L^0{}_\mu dxdydz, \qquad (25)$$

P_μ is called the momentum four-vector, P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy and the integration hyper-surface Σ is described by x^0 = t = constant.

6. Solutions for Szekeres Type I and II Space-times

This section gives us the energy and momentum of the universe based on class II and class I metrics in tele-parallel gravity, respectively.

6.1. Solutions in Szekeres Class II Model

Using the above tetrad and its inverse in Eqs. (5) and (6), we get the following non-vanishing Weitzenböck connection components

$$\begin{aligned}
\Gamma^1{}_{11} &= \tfrac{Q_x}{Q},\\
\Gamma^1{}_{12} &= \tfrac{Q_y}{Q},\\
\Gamma^1{}_{13} &= \tfrac{Q_z}{Q},\\
\Gamma^1{}_{10} &= \tfrac{Q_t}{Q},\\
\Gamma^2{}_{20} &= \tfrac{R_t}{R},\\
\Gamma^3{}_{32} &= \tfrac{h_y}{h},\\
\Gamma^3{}_{30} &= \tfrac{R_t}{R},
\end{aligned} \qquad (26)$$

where x, y, z and t indices describe the derivative with respect to x, y, z and t. The corresponding non-vanishing torsion components are found

$$
\begin{aligned}
T^{1}{}_{12} &= -T^{1}{}_{21} &&= -\tfrac{Q_y}{Q}, \\
T^{1}{}_{13} &= -T^{1}{}_{31} &&= -\tfrac{Q_z}{Q}, \\
T^{1}{}_{10} &= -T^{1}{}_{01} &&= -\tfrac{Q_t}{Q}, \\
T^{2}{}_{20} &= -T^{2}{}_{02} &&= -\tfrac{R_t}{R}, \\
T^{3}{}_{23} &= -T^{3}{}_{32} &&= \tfrac{h_y}{h}, \\
T^{3}{}_{30} &= -T^{3}{}_{03} &&= -\tfrac{R_t}{R},
\end{aligned} \tag{27}
$$

Taking these results into Eq. (19), the non-zero energy components of the tensor $S^{\mu\nu\lambda}$ are found as:

$$
\begin{aligned}
S^{112} &= \tfrac{1}{2}\tfrac{h_y}{Q^2R^2h}, \\
S^{110} &= -\tfrac{R_t}{Q^2R}, \\
S^{223} &= \tfrac{1}{2}\tfrac{Q_z}{R^4h^2Q} \\
S^{220} &= -\tfrac{1}{2}\tfrac{(RQ)_t}{R^3Q} \\
S^{323} &= -\tfrac{1}{2}\tfrac{Q_y}{R^4h^2Q} \\
S^{330} &= -\tfrac{1}{2}\tfrac{(QR)_t}{R^3h^2Q} \\
S^{020} &= \tfrac{1}{2}\tfrac{(Qh)_y}{R^2Qh} \\
S^{030} &= \tfrac{1}{2}\tfrac{Q_z}{R^2h^2Q} \\
S^{002} &= -\tfrac{1}{2}\tfrac{(Qh)_y}{R^2Qh} \\
S^{003} &= -\tfrac{1}{2}\tfrac{Q_z}{R^2h^2Q}
\end{aligned} \tag{28}
$$

Using these components and the Eqs. (15), (16) and (17), we obtain the energy and momentum densities in the sense of Einstein, Bergmann-Thomson and Landau-Lifshitz respectively, as follows

$$
\begin{aligned}
hE_0^0 &= \frac{1}{8\pi}\frac{2hQ_yh_y+h^2Q_{yy}+hQh_{yy}+Q_{zz}}{h},\\
hE_1^0 &= -\frac{RhR_tQ_x}{4\pi},\\
hE_2^0 &= \frac{R(RQ_th_y+hR_tQ_y+QR_th_y+RhQ_{ty})}{8\pi},\\
hE_3^0 &= \frac{hR(Q_zR_t+Q_{tz}R)}{8\pi},
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
hB_0^0 &= \frac{1}{8\pi}\frac{2hQ_yh_y+h^2Q_{yy}+hQh_{yy}+Q_{zz}}{h},\\
hB_1^0 &= -\frac{RhR_tQ_x}{4\pi},\\
hB_2^0 &= \frac{R(RQ_th_y+hR_tQ_y+QR_th_y+RhQ_{ty})}{8\pi},\\
hB_3^0 &= \frac{hR(Q_zR_t+Q_{tz}R)}{8\pi},
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
hL_0^0 &= \frac{1}{8}\frac{R^2(Q_y^2h^2+4QQ_yhh_y+h_y^2Q^2+QQ_{yy}h^2+Q^2hh_{yy}+Q_z^2+QQ_{zz})}{\pi},\\
hL_2^0 &= \frac{1}{8}\frac{R^3h(2Q_yhR_tQ+Q_yhQ_tR+2h_yQ^2R_t+2h_yQRQ_t+hQRQ_{ty})}{\pi},\\
hL_3^0 &= \frac{R^3h^2}{8}\frac{(2Q_zR_tQ+Q_zQ_tR+QQ_{tz}R)}{\pi},
\end{aligned}
\tag{31}
$$

6.2. Solutions in Szekeres Class I Model

Using the above tetrad and its inverse in Eqs. (5) and (6), we get the following non-vanishing Weitzenböck connection components

$$
\begin{aligned}
\Gamma^1{}_{11} &= \Gamma^2{}_{21} &&= B_x, \\
\Gamma^1{}_{12} &= \Gamma^2{}_{22} &&= B_y, \\
\Gamma^1{}_{13} &= \Gamma^2{}_{23} &&= B_z, \\
\Gamma^1{}_{10} &= \Gamma^2{}_{20} &&= B_t, \\
\Gamma^3{}_{31} &= A_x, \\
\Gamma^3{}_{32} &= A_y, \\
\Gamma^3{}_{33} &= A_z, \\
\Gamma^3{}_{34} &= A_t,
\end{aligned}
\tag{32}
$$

where x, y, z and t indices describe the derivative with respect to x, y, z and t. The corresponding non-vanishing torsion components are found

$$
\begin{aligned}
T^1_{12} &= -T^1_{21} &&= -B_y, \\
T^1_{13} &= -T^1_{31} &&= T^2_{23} &&= -T^2_{32} &&= -B_z, \\
T^1_{10} &= -T^1_{01} &&= T^2_{20} &&= -T^2_{02} &&= -B_t, \\
T^2_{21} &= -T^2_{12} &&= -B_x, \\
T^3_{13} &= -T^3_{31} &&= -A_x, \\
T^3_{23} &= -T^3_{32} &&= A_y, \\
T^3_{30} &= -T^3_{03} &&= -A_t,
\end{aligned}
\tag{33}
$$

Taking these results into Eq. (19), the non-zero energy components of the tensor $S^{\mu\nu\lambda}$ are found as:

$$
\begin{aligned}
S^{112} &= \tfrac{1}{2}e^{(-4B)}A_y,\\
S^{113} = S^{223} &= \tfrac{1}{2}B_z e^{-2(A+B)},\\
S^{110} = S^{220} &= -\tfrac{1}{2}e^{(-2B)}(B_t + A_t)\\
S^{212} &= -\tfrac{1}{2}e^{(-4B)}A_x\\
S^{313} &= -\tfrac{1}{2}B_x e^{-2(A+B)}\\
S^{323} &= -\tfrac{1}{2}B_y e^{-2(A+B)}\\
S^{330} &= -e^{(-2A)}B_t\\
S^{010} &= \tfrac{1}{2}(B_x + A_x)e^{-2B}\\
S^{020} &= \tfrac{1}{2}(B_y + A_y)e^{-2B}\\
S^{003} &= -\tfrac{1}{2}\tfrac{Q_z}{R^2h^2Q}
\end{aligned}
\tag{34}
$$

Using these components and the Eqs. (15), (16) and (17) we obtain the energy and momentum densities in the sense of Einstein, Bergmann-Thomson and Landau-Lifshitz respectively, as follows

$$
\begin{aligned}
hE^0_0 &= \tfrac{1}{8\pi}[e^A(B_xA_x + A_x^2 + B_{xx} + A_{xx} + A_yB_y + A_y^2 + B_{yy} + A_{yy})\\
&\quad +2e^{2B-A}(2B_z^2 - B_zA_z + B_{zz})],\\
hE^0_1 &= \tfrac{1}{8\pi}[e^{(2B+A)}(A_xB_t + A_xA_t + B_{tx} + A_{tx})],\\
hE^0_2 &= \tfrac{1}{8\pi}[e^{(2B+A)}(B_tA_y + A_yA_t + B_{ty} + A_{ty})],\\
hE^0_3 &= -\tfrac{1}{4\pi}[e^{(2B+A)}(B_tA_z - 2B_zB_t - B_{tz})],
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
hB_0^0 &= \tfrac{1}{8\pi}[e^A(B_xA_x + A_x^2 + B_{xx} + A_{xx} + A_yB_y + A_y^2 + B_{yy} + A_{yy}) \\
&\quad +2e^{2B-A}(2B_z^2 - B_zA_z + B_{zz})], \\
hB_1^0 &= \tfrac{1}{8\pi}[e^{(2B+A)}(A_xB_t + A_xA_t + B_{tx} + A_{tx})], \\
hB_2^0 &= \tfrac{1}{8\pi}[e^{(2B+A)}(B_tA_y + A_yA_t + B_{ty} + A_{ty})], \\
hB_3^0 &= -\tfrac{1}{4\pi}[e^{(2B+A)}(B_tA_z - 2B_zB_t - B_{tz})],
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
hL_0^0 &= \tfrac{1}{8\pi}[e^{2(B+A)}(B_{xx} + 2B_x^2 + 4B_xA_x + A_{xx} + 2A_x^2 + B_{yy} + 2B_y^2 + \\
&\quad 4B_yA_y + A_{yy} + 2A_y^2) + e^{4B}(8B_z^2 + 2B_{zz})], \\
hL_1^0 &= \tfrac{e^{2(A+2B)}}{8}\tfrac{(2B_xB_t + 2A_tB_x + 2A_xB_t + 2A_xA_t + B_{tx} + A_{tx})}{\pi}, \\
hL_2^0 &= \tfrac{e^{2(A+2B)}}{8}\tfrac{(2B_yB_t + 2A_tB_y + 2B_tA_y + 2A_yA_t + B_{ty} + A_{ty})}{\pi}, \\
hL_3^0 &= \tfrac{e^{2(A+2B)}}{4}\tfrac{(4B_zB_t + B_{tz})}{\pi},
\end{aligned} \tag{37}
$$

This results agree with the previous results obtained by Aygün *et al.* [50] in general relativity by using these different energy-momentum complexes.

7. Summary and Discussion

The subject of energy-momentum localization in the general theory of relativity and tele-parallel gravity has been very exciting and interesting; however it has been associated with some debate. Recently, some researchers have been interested in studying the energy content of the universe in various models.

The main propose of present paper is to show that it is possible to solve the problem of localization of energy in tele-parallel gravity by using the energy and momentum complexes. In this paper, we get the energy distributions of the inhomogeneous and anisotropic Szekeres cosmological models, we have considered three different energy and momentum complexes in tele-parallel gravity: e.g. Bergmann-Thomson, Einstein and Landau-Lifshitz. We found that; (i) the total energy and momentum (due to matter plus field) distribution in tele-parallel gravity for Einstein and Bergmann-Thomson formulations are exactly same in Szekeres class II type space-time and different definitions of this formulation agree with each other. (ii) but we also find that the energy-momentum prescription of Landau-Lifshitz disagree in tele-parallel gravity with these definitions. (iii) The total energy and momentum (due to matter plus field) distribution in tele-parallel gravity for Einstein and Bergmann-Thomson formulations are exactly same in Szekeres class I type space-time. (iv) But we also find that the energy-momentum prescription of Landau-Lifshitz disagree in Szekeres class I space time with these definitions. (v) This results are the same as a previous works of Aygün *et al.* [50]. The authors investigated the same problem in general relativity by using Einstein, Bergman-Thomson, Møller and Landau-Lifshitz (LL) energy-momentum complexes and found same results in Szekeres class I and II space times. (vi) From this point of view; both general relativity and tele-parallel gravity are equivalent theories, that is the energy and momentum densities are the same, using different energy-momentum complexes, in both theories. (vii) In general relativity, except energy distribution ($\Theta_0^0 = \Xi_0^0$) other components disagree each other in Szekeres type I space-time for Einstein and Bergmann-Thomson formulations, but in tele-parallel gravity, all components agree each other. (viii) This results advocate the importance of tele-parallel gravity.

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ENERGY AND MOMENTUM OF THE SZEKERES UNIVERSES IN TELE-PARALLEL GRAVITY

S. AYGÜN†, İ. TARHAN†, H. BAYSAL‡

†Department of Physics, Art and Science Faculty, Çanakkale Onsekiz Mart University, 17020 Çanakkale, Turkey

†Çanakkale Onsekiz Mart University, Astrophysics Research Center, 17020 Çanakkale, Turkey

‡Department of Secondary Science and Mathematics Education, Faculty of Education, Çanakkale Onsekiz Mart University, 17020 Çanakkale, Turkey

In order to evaluate the energy distribution (due to matter and fields including gravitation) associated with a space-time model of Szekeres class I and II metrics, we consider the Einstein, Bergmann-Thomson and Landau-Lifshitz energy and/or momentum definitions in the tele-parallel gravity (the tetrad theory of gravitation). We find the same energy distribution using Einstein and Bergmann-Thomson formulations, but we also find that the energy-momentum prescription of Landau-Lifshitz disagree in general with these definitions. This results are the same as a previous works of Aygün *et al.*, they investigated the same problem in general relativity by using Einstein, Bergman-Thomson, Møller and Landau-Lifshitz (LL) energy-momentum complexes.

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1. Introduction

The issue of energy localization was first discussed during the early years after the development of general relativity and debate continued for decades. There are different attempts to find a general accepted definition of the energy density for the gravitational field. However, there is still no generally accepted definition known. The foremost endeavor was made by Einstein [1] who suggested a definition for energy-momentum distribution. Following this definition, many physicists proposed different energy-momentum complexes: e.g. Tolman [2], Landau and Lifshitz [3], Papapetrou [4], Bergmann and Thomson [5], Weinberg [6], Qadir and Sharif [7] and Møller [8]. Except for the Møller definition, others are restricted to calculate the energy and momentum distributions in quasi-Cartesian coordinates to get a reasonable and meaningful result.

Despite these drawbacks, some interesting results obtained recently leads to the conclusion that these definitions give exactly the same energy distribution for any given space-time [9]-[27]. However, some examples of space-times have been explored which do not support these results [28]-[32].

The problem of energy-momentum localization can also be reformulated in the context of tele-parallel gravity [33, 34, 35]. By working in the context of tele-parallel gravity, Vargas [35] obtained the tele-parallel version of both Einstein and Landau-Lifshitz energy-momentum complexes. He used these definitions and found that the total energy is zero in Friedmann-Robertson-Walker space-time. His results are the same as those calculated in general relativity. Salti and his collaborators [36]-[39] considered different space-times for various definitions in tele-parallel gravity to obtain the energy-momentum distribution in a given model. Their results agree with the previous results obtained in the theory of general relativity.

The basic purpose of this paper is that using the energy-momentum definitions of Einstein, Bergmann-Thomson and Landau-Lifshitz in tele-parallel gravity to obtain the total energy associated with Szekeres type I and II space times. We will proceed according to the following scheme. In the next section,we briefly present the Szekeres Universes. Then, in Sec. III, we give Szekeres type II space-time and its tetrad components. In Sec. IV, we give Szekeres type I space-time and its tetrad components. In Sec. V, we present the energy-momentum definitions of Einstein, Bergmann-Thomson and Landau-Lifshitz in tele-parallel gravity. In Sec. VI gives us the Szekeres class I and class II solutions. Finally, Sec. VII is devoted to concluding remarks. Throughout this paper we choose units such that $G = 1$ and $c = 1$ and follow the convention that indices take values from 0 to 3 otherwise stated.

2. The Szekeres Class I and Szekeres Class II Space-Times

Szekeres [40] derived a remarkable set of inhomogeneous exact solutions of Einstein's field equations without cosmological constant. The source of curvature of the models is an expanding, irrotational, and geodesic dust. These solutions are divided into two classes usually denoted by I and II. The class I solutions are usually presented in a way that is formally analogous to the Tolman-Bondi spherically-symmetric solutions, which they generalize. This class of solutions has primarily been used to model non-spherical collapse of an inhomogeneous dust cloud [41]. The class II solutions are usually considered as generalizations of the Kantowski-Sachs [42] and Friedmann-

Robertson -Walker (FRW) solutions and have primarily been studied as cosmological models [43]. Those of class II are more important as cosmological models, because they can closely approximate, over a finite time interval, the FRW dust models.

In this section, we introduce the Szekeres class II and Szekeres class I metrics and then using these space-times we make some required calculations.

3. The Szekeres Class II Model

The Szekeres class II space-time is defined by the line element [44]

$$ds^2 = -dt^2 + Q^2 dx^2 + R^2(dy^2 + h^2 dz^2). \tag{1}$$

where $Q=Q(x,y,z,t)$, $R=R(t)$ and $h=h(y)$ are functions to be determined. For the line element (1), $g_{\mu\nu}$ is defined by

$$g_{\mu\nu} = -\delta^0_\mu \delta^0_\nu + Q^2 \delta^1_\mu \delta^1_\nu + R^2 \delta^2_\mu \delta^2_\nu + (Rh)^2 \delta^3_\mu \delta^3_\nu \tag{2}$$

and its inverse $g^{\mu\nu}$

$$g^{\mu\nu} = -\delta^\mu_0 \delta^\nu_0 + Q^{-2} \delta^\mu_1 \delta^\nu_1 + R^{-2} \delta^\mu_2 \delta^\nu_2 + (Rh)^{-2} \delta^\mu_3 \delta^\nu_3 \tag{3}$$

The non-trivial tetrad field induces a tele-parallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu \tag{4}$$

Using this relation, we obtain the tetrad components:

$$h^a{}_\mu = \delta^a_0 \delta^0_\mu + Q \delta^a_1 \delta^1_\mu + R \delta^a_2 \delta^2_\mu + Rh \delta^a_3 \delta^3_\mu \tag{5}$$

and its inverse is

$$h_a{}^\mu = \delta^0_a \delta^\mu_0 + Q^{-1} \delta^1_a \delta^\mu_1 + R^{-1} \delta^2_a \delta^\mu_2 + (Rh)^{-1} \delta^3_a \delta^\mu_3 \tag{6}$$

For the line-element which describes the Szekeres Type II space universe, one can introduce the tetrad basis

$$\theta^0 = dt, \qquad \theta^1 = Q dx, \qquad \theta^2 = R dy, \qquad \theta^3 = (Rh) dz. \tag{7}$$

4. The Szekeres Class I Model

The Szekeres class I space-time is defined by the line element

$$ds^2 = -dt^2 + e^{2B}(dx^2 + dy^2) + e^{2A}dz^2. \tag{8}$$

where $A = A(x, y, z, t)$, $B = B(x, y, z, t)$ are functions to be determined.

$$g_{\mu\nu} = -\delta^0_\mu\delta^0_\nu + e^{2B}\delta^1_\mu\delta^1_\nu + e^{2B}\delta^2_\mu\delta^2_\nu + e^{2A}\delta^3_\mu\delta^3_\nu \tag{9}$$

and its inverse $g^{\mu\nu}$

$$g^{\mu\nu} = -\delta^\mu_0\delta^\nu_0 + e^{-2B}\delta^\mu_1\delta^\nu_1 + e^{-2B}\delta^\mu_2\delta^\nu_2 + e^{-2A}\delta^\mu_3\delta^\nu_3 \tag{10}$$

The non-trivial tetrad field induces a tele-parallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab}h^a{}_\mu h^b{}_\nu \tag{11}$$

Using this relation, we obtain the tetrad components:

$$h^a{}_\mu = \delta^a_0\delta^0_\mu + e^B\delta^a_1\delta^1_\mu + e^B\delta^a_2\delta^2_\mu + e^A\delta^a_3\delta^3_\mu \tag{12}$$

and its inverse is

$$h_a{}^\mu = \delta^0_a\delta^\mu_0 + e^{-B}\delta^1_a\delta^\mu_1 + e^{-B}\delta^2_a\delta^\mu_2 + e^{-A}\delta^3_a\delta^\mu_3 \tag{13}$$

For the line-element which describes the Szekeres Type I space universe, one can introduce the tetrad basis

$$\theta^0 = dt, \qquad \theta^1 = e^B dx, \qquad \theta^2 = e^B dy, \qquad \theta^3 = e^A dz. \tag{14}$$

5. Energy-Momentum in Tele-parallel Gravity

The tele-parallel gravity is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry [45]. In the theory of the tele-parallel gravity, gravitation is attributed to torsion [46], which plays the role of a force [47], and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a tele-parallel structure which is directly related to the presence of the gravitational field. The interesting place of tele-parallel gravity is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the situation, for example, in the energy and momentum problem, which becomes more transparent when considered from the tele-parallel point of view.

The Einstein, Bergmann-Thomson and Landau-Lifshitz's energy-momentum complexes in tele-parallel gravity[48] are respectively:

$$hE^{\mu}{}_{\nu} = \frac{1}{4\pi}\partial_{\lambda}(U_{\nu}{}^{\mu\lambda}) \tag{15}$$

$$hB^{\mu\nu} = \frac{1}{4\pi}\partial_{\lambda}(g^{\mu\beta}U_{\beta}{}^{\nu\lambda}) \tag{16}$$

$$hL^{\mu\nu} = \frac{1}{4\pi}\partial_{\lambda}(hg^{\mu\beta}U_{\beta}{}^{\nu\lambda}) \tag{17}$$

where $U_{\beta}{}^{\nu\lambda}$ is the Freud's super-potential, which is given by:

$$U_{\beta}{}^{\nu\lambda} = hS_{\beta}{}^{\nu\lambda}. \tag{18}$$

where $h = \mathtt{det}(h^{a}{}_{\mu})$ and $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = m_1 T^{\mu\nu\lambda} + \frac{m_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{m_3}{2}(g^{\mu\lambda}T^{\beta\nu}_{\ \ \beta} - g^{\nu\mu}T^{\beta\lambda}_{\ \ \beta}) \tag{19}$$

with m_1, m_2 and m_3 the three dimensionless coupling constants of tele-parallel gravity [49]. For the tele-parallel equivalent of general relativity the specific choice of these three constants are:

$$m_1 = \frac{1}{4}, \qquad m_2 = \frac{1}{2}, \qquad m_3 = -1 \tag{20}$$

To calculate this tensor, firstly we must calculate Weitzenböck connection:

$$\Gamma^{\alpha}{}_{\mu\nu} = h_{a}{}^{\alpha}\partial_{\nu}h^{a}{}_{\mu} \tag{21}$$

and torsion of the Weitzenböck connection:

$$T^{\mu}{}_{\nu\lambda} = \Gamma^{\mu}{}_{\lambda\nu} - \Gamma^{\mu}{}_{\nu\lambda} \tag{22}$$

The energy-momentum complexes of Einstein, Bergmann-Thomson and Landau-Lifshitz in the tele-parallel gravity are given by the following equations, respectively,

$$P^{E}_{\mu} = \int_{\Sigma} hE^{0}{}_{\mu}dxdydz, \tag{23}$$

$$P^{B}_{\mu} = \int_{\Sigma} hB^{0}{}_{\mu}dxdydz, \tag{24}$$

$$P_\mu^L = \int_\Sigma hL^0{}_\mu dxdydz, \tag{25}$$

P_μ is called the momentum four-vector, P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy and the integration hyper-surface Σ is described by $x^0 = \mathrm{t} = \mathrm{constant}$.

6. Solutions for Szekeres Type I and II Space-times

This section gives us the energy and momentum of the universe based on class II and class I metrics in tele-parallel gravity, respectively.

6.1. Solutions in Szekeres Class II Model

Using the above tetrad and its inverse in Eqs. (5) and (6), we get the following non-vanishing Weitzenböck connection components

$$\begin{aligned}
\Gamma^1{}_{11} &= \tfrac{Q_x}{Q}, \\
\Gamma^1{}_{12} &= \tfrac{Q_y}{Q}, \\
\Gamma^1{}_{13} &= \tfrac{Q_z}{Q}, \\
\Gamma^1{}_{10} &= \tfrac{Q_t}{Q}, \\
\Gamma^2{}_{20} &= \tfrac{R_t}{R}, \\
\Gamma^3{}_{32} &= \tfrac{h_y}{h}, \\
\Gamma^3{}_{30} &= \tfrac{R_t}{R},
\end{aligned} \tag{26}$$

where x, y, z and t indices describe the derivative with respect to x, y, z and t. The corresponding non-vanishing torsion components are found

$$
\begin{aligned}
T^{1}{}_{12} &= -T^{1}{}_{21} &&= -\tfrac{Q_y}{Q},\\
T^{1}{}_{13} &= -T^{1}{}_{31} &&= -\tfrac{Q_z}{Q},\\
T^{1}{}_{10} &= -T^{1}{}_{01} &&= -\tfrac{Q_t}{Q},\\
T^{2}{}_{20} &= -T^{2}{}_{02} &&= -\tfrac{R_t}{R},\\
T^{3}{}_{23} &= -T^{3}{}_{32} &&= \tfrac{h_y}{h},\\
T^{3}{}_{30} &= -T^{3}{}_{03} &&= -\tfrac{R_t}{R},
\end{aligned} \tag{27}
$$

Taking these results into Eq. (19), the non-zero energy components of the tensor $S^{\mu\nu\lambda}$ are found as:

$$
\begin{aligned}
S^{112} &= \tfrac{1}{2}\tfrac{h_y}{Q^2R^2h},\\
S^{110} &= -\tfrac{R_t}{Q^2R},\\
S^{223} &= \tfrac{1}{2}\tfrac{Q_z}{R^4h^2Q}\\
S^{220} &= -\tfrac{1}{2}\tfrac{(RQ)_t}{R^3Q}\\
S^{323} &= -\tfrac{1}{2}\tfrac{Q_y}{R^4h^2Q}\\
S^{330} &= -\tfrac{1}{2}\tfrac{(QR)_t}{R^3h^2Q}\\
S^{020} &= \tfrac{1}{2}\tfrac{(Qh)_y}{R^2Qh}\\
S^{030} &= \tfrac{1}{2}\tfrac{Q_z}{R^2h^2Q}\\
S^{002} &= -\tfrac{1}{2}\tfrac{(Qh)_y}{R^2Qh}\\
S^{003} &= -\tfrac{1}{2}\tfrac{Q_z}{R^2h^2Q}
\end{aligned} \tag{28}
$$

Using these components and the Eqs. (15), (16) and (17), we obtain the energy and momentum densities in the sense of Einstein, Bergmann-Thomson and Landau-Lifshitz respectively, as follows

$$\begin{aligned}
hE^0_0 &= \frac{1}{8\pi}\frac{2hQ_yh_y+h^2Q_{yy}+hQh_{yy}+Q_{zz}}{h},\\
hE^0_1 &= -\frac{RhR_tQ_x}{4\pi},\\
hE^0_2 &= \frac{R(RQ_th_y+hR_tQ_y+QR_th_y+RhQ_{ty})}{8\pi},\\
hE^0_3 &= \frac{hR(Q_zR_t+Q_{tz}R)}{8\pi},
\end{aligned} \tag{29}$$

$$\begin{aligned}
hB^0_0 &= \frac{1}{8\pi}\frac{2hQ_yh_y+h^2Q_{yy}+hQh_{yy}+Q_{zz}}{h},\\
hB^0_1 &= -\frac{RhR_tQ_x}{4\pi},\\
hB^0_2 &= \frac{R(RQ_th_y+hR_tQ_y+QR_th_y+RhQ_{ty})}{8\pi},\\
hB^0_3 &= \frac{hR(Q_zR_t+Q_{tz}R)}{8\pi},
\end{aligned} \tag{30}$$

$$\begin{aligned}
hL^0_0 &= \frac{1}{8}\frac{R^2(Q_y^2h^2+4QQ_yhh_y+h_y^2Q^2+QQ_{yy}h^2+Q^2hh_{yy}+Q_z^2+QQ_{zz})}{\pi},\\
hL^0_2 &= \frac{1}{8}\frac{R^3h(2Q_yhR_tQ+Q_yhQ_tR+2h_yQ^2R_t+2h_yQRQ_t+hQRQ_{ty})}{\pi},\\
hL^0_3 &= \frac{R^3h^2}{8}\frac{(2Q_zR_tQ+Q_zQ_tR+QQ_{tz}R)}{\pi},
\end{aligned} \tag{31}$$

6.2. Solutions in Szekeres Class I Model

Using the above tetrad and its inverse in Eqs. (5) and (6), we get the following non-vanishing Weitzenböck connection components

$$
\begin{aligned}
\Gamma^1{}_{11} &= \Gamma^2{}_{21} &&= B_x, \\
\Gamma^1{}_{12} &= \Gamma^2{}_{22} &&= B_y, \\
\Gamma^1{}_{13} &= \Gamma^2{}_{23} &&= B_z, \\
\Gamma^1{}_{10} &= \Gamma^2{}_{20} &&= B_t, \\
\Gamma^3{}_{31} &= A_x, \\
\Gamma^3{}_{32} &= A_y, \\
\Gamma^3{}_{33} &= A_z, \\
\Gamma^3{}_{34} &= A_t,
\end{aligned} \tag{32}
$$

where x, y, z and t indices describe the derivative with respect to x, y, z and t. The corresponding non-vanishing torsion components are found

$$
\begin{aligned}
T^1_{12} &= -T^1_{21} &&= -B_y, \\
T^1_{13} &= -T^1_{31} &&= T^2_{23} &&= -T^2_{32} &&= -B_z, \\
T^1_{10} &= -T^1_{01} &&= T^2_{20} &&= -T^2_{02} &&= -B_t, \\
T^2_{21} &= -T^2_{12} &&= -B_x, \\
T^3_{13} &= -T^3_{31} &&= -A_x, \\
T^3_{23} &= -T^3_{32} &&= A_y, \\
T^3_{30} &= -T^3_{03} &&= -A_t,
\end{aligned} \tag{33}
$$

Taking these results into Eq. (19), the non-zero energy components of the tensor $S^{\mu\nu\lambda}$ are found as:

$$
\begin{aligned}
S^{112} &= \tfrac{1}{2}e^{(-4B)}A_y, \\
S^{113} = S^{223} &= \tfrac{1}{2}B_z e^{-2(A+B)}, \\
S^{110} = S^{220} &= -\tfrac{1}{2}e^{(-2B)}(B_t + A_t) \\
S^{212} &= -\tfrac{1}{2}e^{(-4B)}A_x \\
S^{313} &= -\tfrac{1}{2}B_x e^{-2(A+B)} \\
S^{323} &= -\tfrac{1}{2}B_y e^{-2(A+B)} \\
S^{330} &= -e^{(-2A)}B_t \\
S^{010} &= \tfrac{1}{2}(B_x + A_x)e^{-2B} \\
S^{020} &= \tfrac{1}{2}(B_y + A_y)e^{-2B} \\
S^{003} &= -\tfrac{1}{2}\tfrac{Q_z}{R^2 h^2 Q}
\end{aligned}
\tag{34}
$$

Using these components and the Eqs. (15), (16) and (17) we obtain the energy and momentum densities in the sense of Einstein, Bergmann-Thomson and Landau-Lifshitz respectively, as follows

$$
\begin{aligned}
hE_0^0 &= \tfrac{1}{8\pi}[e^A(B_xA_x + A_x^2 + B_{xx} + A_{xx} + A_yB_y + A_y^2 + B_{yy} + A_{yy}) \\
&\quad +2e^{2B-A}(2B_z^2 - B_zA_z + B_{zz})], \\
hE_1^0 &= \tfrac{1}{8\pi}[e^{(2B+A)}(A_xB_t + A_xA_t + B_{tx} + A_{tx})], \\
hE_2^0 &= \tfrac{1}{8\pi}[e^{(2B+A)}(B_tA_y + A_yA_t + B_{ty} + A_{ty})], \\
hE_3^0 &= -\tfrac{1}{4\pi}[e^{(2B+A)}(B_tA_z - 2B_zB_t - B_{tz})],
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
hB^0_0 = \tfrac{1}{8\pi}[e^{A}(B_xA_x + A^2_x + B_{xx} + A_{xx} + A_yB_y + A^2_y + B_{yy} + A_{yy})\\
+2e^{2B-A}(2B^2_z - B_zA_z + B_{zz})],
\end{aligned}
$$

$$
hB^0_1 = \tfrac{1}{8\pi}[e^{(2B+A)}(A_xB_t + A_xA_t + B_{tx} + A_{tx})],
$$

$$
hB^0_2 = \tfrac{1}{8\pi}[e^{(2B+A)}(B_tA_y + A_yA_t + B_{ty} + A_{ty})],
\tag{36}
$$

$$
hB^0_3 = -\tfrac{1}{4\pi}[e^{(2B+A)}(B_tA_z - 2B_zB_t - B_{tz})],
$$

$$
\begin{aligned}
hL^0_0 = \tfrac{1}{8\pi}[e^{2(B+A)}(B_{xx} + 2B^2_x + 4B_xA_x + A_{xx} + 2A^2_x + B_{yy} + 2B^2_y +\\
4B_yA_y + A_{yy} + 2A^2_y) + e^{4B}(8B^2_z + 2B_{zz})],
\end{aligned}
$$

$$
hL^0_1 = \tfrac{e^{2(A+2B)}}{8}\tfrac{(2B_xB_t+2A_tB_x+2A_xB_t+2A_xA_t+B_{tx}+A_{tx})}{\pi},
$$

$$
hL^0_2 = \tfrac{e^{2(A+2B)}}{8}\tfrac{(2B_yB_t+2A_tB_y+2B_tA_y+2A_yA_t+B_{ty}+A_{ty})}{\pi},
$$

$$
hL^0_3 = \tfrac{e^{2(A+2B)}}{4}\tfrac{(4B_zB_t+B_{tz})}{\pi},
\tag{37}
$$

This results agree with the previous results obtained by Aygün *et al.* [50] in general relativity by using these different energy-momentum complexes.

7. Summary and Discussion

The subject of energy-momentum localization in the general theory of relativity and tele-parallel gravity has been very exciting and interesting; however it has been associated with some debate. Recently, some researchers have been interested in studying the energy content of the universe in various models.

The main propose of present paper is to show that it is possible to solve the problem of localization of energy in tele-parallel gravity by using the energy and momentum complexes. In this paper, we get the energy distributions of the inhomogeneous and anisotropic Szekeres cosmological models, we have considered three different energy and momentum complexes in tele-parallel gravity: e.g. Bergmann-Thomson, Einstein and Landau-Lifshitz. We found that; (i) the total energy and momentum (due to matter plus field) distribution in tele-parallel gravity for Einstein and Bergmann-Thomson formulations are exactly same in Szekeres class II type space-time and different definitions of this formulation agree with each other. (ii) but we also find that the energy-momentum prescription of Landau-Lifshitz disagree in tele-parallel gravity with these definitions. (iii) The total energy and momentum (due to matter plus field) distribution in tele-parallel gravity for Einstein and Bergmann-Thomson formulations are exactly same in Szekeres class I type space-time. (iv) But we also find that the energy-momentum prescription of Landau-Lifshitz disagree in Szekeres class I space time with these definitions. (v) This results are the same as a previous works of Aygün *et al.* [50]. The authors investigated the same problem in general relativity by using Einstein, Bergman-Thomson, Møller and Landau-Lifshitz (LL) energy-momentum complexes and found same results in Szekeres class I and II space times. (vi) From this point of view; both general relativity and tele-parallel gravity are equivalent theories, that is the energy and momentum densities are the same, using different energy-momentum complexes, in both theories. (vii) In general relativity, except energy distribution ($\Theta_0^0 = \Xi_0^0$) other components disagree each other in Szekeres type I space-time for Einstein and Bergmann-Thomson formulations, but in tele-parallel gravity, all components agree each other. (viii) This results advocate the importance of tele-parallel gravity.

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